Measurement-driven quantum evolution from a known state

Luis Roa,[1] G. A. Olivares-Rentería,[1] M. L. Ladrón de Guevara,[2] and A. Delgado[1]
[1] *Center for Quantum Optics and Quantum Information, Departamento de Física, Universidad de Concepción, Casilla 160-C, Concepción, Chile.*
[2] *Departamento de Física, Universidad Católica del Norte, Casilla 1280, Antofagasta, Chile.*
(Dated: February 1, 2008)

We study the problem of driving a known initial quantum state onto a known pure state without using a unitary evolution. This task can be achieved by means of von Neumann measurement processes, introducing N observables which are consecutively measured in order to approach the state of the system to the target state. We proved that the probability of projecting onto the target state can be increased meaningfully by adding suitable observables to the process, that is, it converges to 1 when N increases. We also discuss a physical implementation of this scheme.

PACS numbers: 03.67.-a, 03.65.-w

I. INTRODUCTION

The problem of controlling quantum systems has been a renewed subject of study. Quantum computing is basesed on the existence of a set of universal quantum gates which, concatenated, allow one to implement any unitary transformation within a fixed level of accuracy. These quantum gates are implemented through the controlled manipulation of the interactions among different physical systems. Quantum communication protocols, such as quantum teleportation [3], entanglement swapping [4] and dense coding, also require the precise application of some unitary transformations in a finite set of transformations. A related problem has also been addressed in the contex of quantum control [18]. There, the goal is to drive the evolution of an initial, possibly mixed, state to a state having a predetermined expectation value of some observable. This evolution is also considered to be unitary.

In this article we study the control of quantum systems in the case where it is not possible to resort to unitary transformations. Our main goal is to map a *known* quantum state onto another known state via a sequence of measurements with the highest possible success probability, that is, a controlled evolution via measurements only. It has been shown [19] that the mapping of an *unknown* quantum state onto a known pure state can be optimally implemented with the help of two observables only. In this case, the highest success probability is achieved when the eigenstates of the two observables define mutually unbiased bases. It has also been shown that, when the system subjected to the measurements is affected by a decoherence mechanism, only one observable is required [20].

Here we study the case of driving by von Neumann processes [21] a known initial state makeing use of more than two observables. First we analyze the problem of two observables. Thereafter, we show that a new observable can be added in order to achieve a further increase in the success probability. By means of numerical simulations we show that the success probability rapidly approaches the unity when the number of observables increases.

II. DRIVING THE EVOLUTION BY TWO OBSERVABLES

Let us start by supposing that a quantum system is in a *known* ρ state. Our goal is to drive the system to the *known* $|\zeta\rangle$ target state by measurements only. If we measure the $\hat{\zeta}$ observable, whose eigenstates are $\{|\zeta\rangle, |\zeta_\perp\rangle\}$, the probability of projecting to the $|\zeta\rangle$ target state is $p_d = \langle\zeta|\rho|\zeta\rangle$. Natural questions arise: is it possible to increase this direct probability p_d by making use of an intermediate measurement of another observable $\hat{\theta}$? And, if it is possible, then how is the relation among ρ, $\hat{\zeta}$, and $\hat{\theta}$ which maximizes such probability?

So, in order to approach the state of the system to the $|\zeta\rangle$ target state [22], we first measure an observable $\hat{\theta}$ which has the $\{|0_1\rangle, |1_1\rangle\}$ eigenstates. As a second step we perform a measurement of $\hat{\zeta}$. Thus, the probability of reaching the $|\zeta\rangle$ target through one eigenstate of $\hat{\theta}$ followed by a measurement of $\hat{\zeta}$ is given by

$$p_{1,s} = \langle 0_1|\rho|0_1\rangle|\langle 0_1|\zeta\rangle|^2 + \langle 1_1|\rho|1_1\rangle|\langle 1_1|\zeta\rangle|^2. \quad (1)$$

Making use of the normalization of ρ and $|\zeta\rangle$, and of the orthonomalization of $|0_1\rangle$ and $|1_1\rangle$ the previous expression can be cast in the form

$$p_{1,s} = \langle\zeta|\rho|\zeta\rangle - 2\,|\langle 1_1|\zeta\rangle\langle\zeta|0_1\rangle|^2\,(\langle\zeta|\rho|\zeta\rangle - \langle\zeta_\perp|\rho|\zeta_\perp\rangle) + (2|\langle 0_1|\zeta\rangle|^2 - 1)\,[\langle 0_1|\zeta\rangle\langle\zeta_\perp|0_1\rangle\langle\zeta|\rho|\zeta_\perp\rangle + \text{c.c.}]\,. \quad (2)$$

The second term at the r.h.s. of Eq. (2) contributes to increase $p_{1,s}$ with respect to p_d when $\langle\zeta_\perp|\rho|\zeta_\perp\rangle$ is higher than $\langle\zeta|\rho|\zeta\rangle$, otherwise it helps to decrease $p_{1,s}$ with respect to p_d. Meanwhile, the third term plays a role when the ρ initial state has non-diagonal elements different from zero in the $\hat{\zeta}$ representation. If $\rho = I/2$, being I the identity, then $p_{1s} = 1/2$, so that $p_{1,s}$ is independent of the choice of $\hat{\theta}$. If the ρ initial state is diagonal in the $\hat{\zeta}$ representation, then, when $\langle\zeta|\rho|\zeta\rangle < \langle\zeta_\perp|\rho|\zeta_\perp\rangle$, it requires a $\hat{\theta}$ observable unbiased to $\hat{\zeta}$ in order to optimize

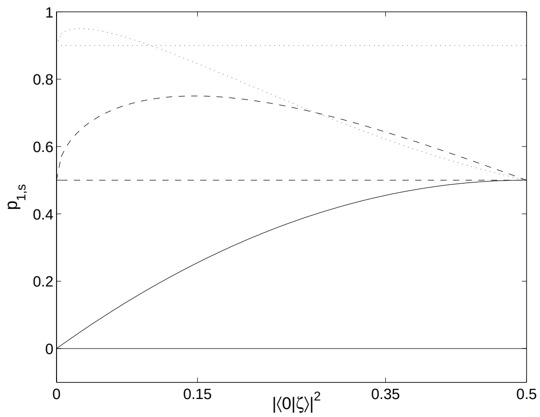

FIG. 1: Probability of success $p_{1,s}$ as a function of $|\langle 0_1|\zeta\rangle|^2$ with $\langle\zeta|\rho|\zeta\rangle = 0$ (solid), $\langle\zeta|\rho|\zeta\rangle = 0.5$ (dash), and $\langle\zeta|\rho|\zeta\rangle = 0.9$ (dot). Horizontal lines are the respective p_d.

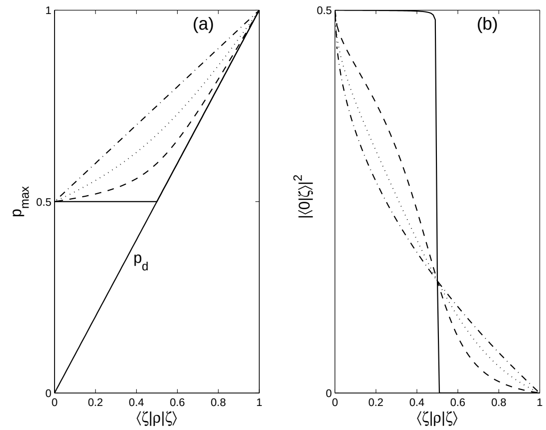

FIG. 2: (a) maximum probability of success p_{max} as a function of $\langle\zeta|\rho|\zeta\rangle$ for different γ values: $\gamma = 1$ (dot-dash), $\gamma = 0.7$ (dot), $\gamma = 0.4$ (dash), and $\gamma = 0$ (solid). (b) $|\langle 0_1|\zeta\rangle|^2$ component as a function of $\langle\zeta|\rho|\zeta\rangle$ for different γ values: $\gamma = 1$ (dot-dash), $\gamma = 0.7$ (dot), $\gamma = 0.4$ (dash), and $\gamma = 0$ (solid).

the process, whereas when $\langle\zeta|\rho|\zeta\rangle > \langle\zeta_\perp|\rho|\zeta_\perp\rangle$, there is not any $\hat{\theta}$ observable which allows to increase $p_{1,s}$ over the value of p_d.

The third term of the r.h.s. of Eq. (2) contributes maximally to $p_{1,s}$ when $\arg(\langle\zeta|\rho|\zeta_\perp\rangle\langle\zeta_\perp|0_1\rangle\langle 0_1|\zeta\rangle) = 0$ and $|\langle 0_1|\zeta\rangle|^2 \geq 1/2$, or $\arg(\langle\zeta|\rho|\zeta_\perp\rangle\langle\zeta_\perp|0_1\rangle\langle 0_1|\zeta\rangle) = \pi$ and $|\langle 0_1|\zeta\rangle|^2 \leq 1/2$. Since both cases are symmetric with respect to $|\langle 0_1|\zeta\rangle|^2 = 1/2$, in the following we consider only the latter.

Figure 1 shows $p_{1,s}$ as a function of $|\langle 0_1|\zeta\rangle|^2$ for different initial values, say: $\langle\zeta|\rho|\zeta\rangle = 0$ (solid line), $\langle\zeta|\rho|\zeta\rangle = 0.5$ (dashed line), and $\langle\zeta|\rho|\zeta\rangle = 0.9$ (dotted line). In all of these cases we have considered complete initial coherence, this is, $|\langle\zeta|\rho|\zeta_\perp\rangle| = \sqrt{\langle\zeta|\rho|\zeta\rangle\langle\zeta_\perp|\rho|\zeta_\perp\rangle}$. The horizontal lines are the respective $p_d = \langle\zeta|\rho|\zeta\rangle$. We can see that for the considered initial conditions there is an interval of $|\langle 0_1|\zeta\rangle|^2$ where $p_{1,s}$ is higher than its associated p_d, and there is a particular value of $|\langle 0_1|\zeta\rangle|^2$ for which $p_{1,s}$ is maximum.

Let us examine what happen for a more general initial condition, i.e., a ρ initial state with $0 \leq |\langle\zeta|\rho|\zeta_\perp\rangle| \leq \sqrt{\langle\zeta|\rho|\zeta\rangle\langle\zeta_\perp|\rho|\zeta_\perp\rangle}$, where $\langle\zeta|\rho|\zeta\rangle \neq 1$. We look for conditions under which the probability of success of the measurement process $M(\zeta)M(\theta)$ is maximum. Optimizing Eq. (2) with respect to $|\langle 0_1|\zeta\rangle|^2$, one finds that the maximum value p_{max} of $p_{1,s}$ is

$$p_{max} = \frac{\langle\zeta|\rho|\zeta\rangle}{2} + \frac{1}{4}(1+R), \tag{3}$$

with

$$R = \sqrt{(1-\gamma^2)(2\langle\zeta|\rho|\zeta\rangle - 1)^2 + \gamma^2}, \tag{4}$$

where we have defined the γ coefficient by the equality

$$|\langle\zeta|\rho|\zeta_\perp\rangle| = \gamma\sqrt{\langle\zeta|\rho|\zeta\rangle\langle\zeta_\perp|\rho|\zeta_\perp\rangle}, \quad 0 \leq \gamma \leq 1,$$

Fig. (2.a) shows the maximum probability (3) as a function of $\langle\zeta|\rho|\zeta\rangle$ for different values of γ: $\gamma = 1$ (dot-dashed line), $\gamma = 0.7$ (dotted line), $\gamma = 0.4$ (dashed line), and $\gamma = 0$ (solid line). The diagonal solid line corresponds to p_d. Notice that for all $\gamma \neq 0$ the optimal probability p_{max} exceeds p_d for all values of $\langle\zeta|\rho|\zeta\rangle$. Also, larger values of γ result in larger values of p_{max}.

The $|0_1\rangle$ eigenstate of the $\hat{\theta}$ observable which optimizes $p_{1,s}$ has a component on the $|\zeta\rangle$ target state given by

$$|\langle 0_1|\zeta\rangle|^2 = \frac{1}{2}\left(1 - \frac{1}{\sqrt{2}}\sqrt{1 + \frac{2\langle\zeta|\rho|\zeta\rangle - 1}{R}}\right). \tag{5}$$

Fig. (2.b) shows the square module (5) of the $|0_1\rangle$ state component onto the target state $|\zeta\rangle$ as a function of the initial probability of the $|\zeta\rangle$ state for different γ values.

When the initial state is pure ($\gamma = 1$), $\rho = |\psi\rangle\langle\psi|$, the square module of the $|0_1\rangle$ state component on the target state $|\zeta\rangle$ becomes

$$|\langle 0_1|\zeta\rangle|^2 = \frac{1 - |\langle\psi|\zeta\rangle|}{2}, \tag{6}$$

which is a linear relation between a square module and a module of two probability amplitudes. When the initial state is pure, $|\psi\rangle$, the average of (3) on the Hilbert space reaches the value $3/4$.

Thus, we have found the $\hat{\theta}$ observable which optimizes the fidelity or the probability of taking the initial known state ρ to the target $|\zeta\rangle$ by means of von Neumann measurements only.

III. PROCESSING BY N+1 OBSERVABLES

First of all we study the case where three observables are implemented in order to achieve the target. Hence the conlusions obtained are easily generalized when $N+1$ observables are considered.

Now we suppose that before measuring the observable $\hat{\zeta}$ we measure two observables, say $\hat{\theta}_1$ followed by $\hat{\theta}_2$, which define orthonormal bases $\{|0_j\rangle, |1_j\rangle\}$, $j = 1, 2$, respectively. In other words, we shall apply three consecutive von Neumann measurement processes, first $M(\hat{\theta}_1)$ followed by $M(\hat{\theta}_2)$ and finally by $M(\hat{\zeta})$, which shall be denoted by the simple product $M(\hat{\zeta})M(\hat{\theta}_2)M(\hat{\theta}_1)$. The probability of driving the known initial state ρ toward the $|\zeta\rangle$ target, by means of the von Neumann measurement $M(\hat{\zeta})M(\hat{\theta}_2)M(\hat{\theta}_1)$ process, is given by

$$\begin{aligned} p_{2,s} &= 1 - \langle 0_1|\rho|0_1\rangle - (1 - 2\langle 0_1|\rho|0_1\rangle) \\ &\times \left[1 - |\langle 0_2|\zeta\rangle|^2 - |\langle 0_1|0_2\rangle|^2 \left(1 - 2\,|\langle 0_2|\zeta\rangle|^2\right)\right], \end{aligned} \tag{7}$$

where the quantities $\langle 0_1|\rho|0_1\rangle$ and $|\langle 0_1|0_2\rangle|^2$ entering in $p_{2,s}$ are considered to be functions of the coefficients of ρ in the basis of the ζ observable, of the quantities $|\langle 0_1|\zeta\rangle|^2$ and $|\langle 0_2|\zeta\rangle|^2$, and of the phases φ and ϕ of $\langle\zeta|\rho|\zeta_\perp\rangle\langle 0_1|\zeta\rangle\langle\zeta_\perp|0_1\rangle$ and $\langle 0_1|\zeta\rangle\langle\zeta|0_2\rangle\langle 0_1|\zeta_\perp\rangle\langle\zeta_\perp|0_2\rangle$, respectively.

The problem of optimizing the probability $p_{2,s}$, Eq. (7), leads to a set of nonlinear equations for the quantities $|\langle 0_1|\zeta\rangle|^2$, $|\langle 0_2|\zeta\rangle|^2$, φ and ϕ, which can not be analytically solved. However, we are able to show that, under certain conditions, it is possible to choose the observable θ_2 in such a way that $p_{2,s}$ becomes higher than $p_{1,s}$.

The probability $p_{s,2}$ can be also written as

$$\begin{aligned} p_{2,s} &= p_{1,s} + \langle 0_1|\rho|0_1\rangle \left(|\langle 0_1|0_2\rangle|^2\,|\langle 0_2|\zeta\rangle|^2\right. \\ &\quad \left. + |\langle 0_1|1_2\rangle|^2\,|\langle 1_2|\zeta\rangle|^2 - |\langle 0_1|\zeta\rangle|^2\right) \\ &\quad + \langle 1_1|\rho|1_1\rangle \left(|\langle 1_1|0_2\rangle|^2\,|\langle 0_2|\zeta\rangle|^2\right. \\ &\quad \left. + |\langle 1_1|1_2\rangle|^2\,|\langle 1_2|\zeta\rangle|^2 - |\langle 1_1|\zeta\rangle|^2\right), \end{aligned} \tag{8}$$

where $p_{1,s}$ is given by Eq. (1). Hence $p_{2,s}$ is higher than $p_{1,s}$ under the conditions:

$$\langle 0_1|\rho|0_1\rangle > \langle 1_1|\rho|1_1\rangle, \tag{9}$$

and

$$|\langle 0_1|0_2\rangle|^2\,|\langle 0_2|\zeta\rangle|^2 + |\langle 0_1|1_2\rangle|^2\,|\langle 1_2|\zeta\rangle|^2 > |\langle 0_1|\zeta\rangle|^2\,. \tag{10}$$

The condition (10) means that the basis $\{|0_2\rangle, |1_2\rangle\}$ has to be chosen in a way such that the probability of taking the state $|0_1\rangle$ to the state $|\zeta\rangle$ by means of the $M(\zeta)M(\theta_2)$ process be higher than the probability of taking the state $|0_1\rangle$ to the state $|\zeta\rangle$ by means of the $M(\zeta)$ process. We have already shown, in section II, that such a choice is always possible. The probability $p_{2,s}$ is higher than $p_{1,s}$ also under the conditions:

$$\langle 0_1|\rho|0_1\rangle < \langle 1_1|\rho|1_1\rangle, \tag{11}$$

and

$$|\langle 1_1|0_2\rangle|^2\,|\langle 0_2|\zeta\rangle|^2 + |\langle 1_1|1_2\rangle|^2\,|\langle 1_2|\zeta\rangle|^2 > |\langle 1_1|\zeta\rangle|^2\,. \tag{12}$$

The latter condition has the same meaning as the (10) inequality, but in this case starting from the $|1_1\rangle$ state instead of from the $|0_1\rangle$ state. This condition can also be always satisfied.

The above result can be generalized to the case of N observables $theta_i$. In this case we suppose that, before measuring the observable $\hat{\zeta}$, we measure N observables, say $\hat{\theta}_1$, $\hat{\theta}_2$, ..., $\hat{\theta}_N$, each one defining an orthonormal basis $\{|i_j\rangle\}$ respectively, with $i_j = 0, 1$ and $j = 1, 2\ldots, N$. The probability of driving the known initial state ρ towards the $|\zeta\rangle$ target, by means of the von Neumann measurement processes $M(\hat{\zeta})M(\hat{\theta}_N)\ldots M(\hat{\theta}_2)M(\hat{\theta}_1)$, can be calculated recursively as

$$p_{N,s} = \langle\zeta|\rho_N|\zeta\rangle, \tag{13}$$

where ρ_N is given by

$$\rho_N = \sum_{i_N=0}^{1} \langle i_N|\rho_{N-1}|i_N\rangle |i_N\rangle\langle i_N|. \tag{14}$$

The difference $\Delta = p_{N+1,s} - p_{N,s}$ can be read as

$$\begin{aligned} \Delta &= \sum_{i_N=0}^{1} \langle i_N|\rho_{N-1}|i_N\rangle \\ &\times \Big(\sum_{i_{N+1}=0}^{1} |\langle i_N|i_{N+1}\rangle|^2 |\langle i_{N+1}|\zeta\rangle|^2 - |\langle i_N|\zeta\rangle|^2\Big). \end{aligned} \tag{15}$$

The positivity of this difference is guaranteed under the conditions

$$\langle 0_N|\rho_{N-1}|0_N\rangle > \langle 1_N|\rho_{N-1}|1_N\rangle, \tag{16}$$

and

$$\sum_{i_{N+1}=0}^{1} |\langle 0_N|i_{N+1}\rangle|^2 |\langle i_{N+1}|\zeta\rangle|^2 > |\langle 0_N|\zeta\rangle|^2. \tag{17}$$

The latter condition means that the basis $\{|0_{N+1}\rangle, |1_{N+1}\rangle\}$ must be chosen in a way such that the probability of taking state $|0_N\rangle$ to state $|\zeta\rangle$ by means of the $M(\zeta)M(\theta_{N+1})$ process be higher than the probability obtained by means of the $M(\zeta)$ process. In section II we have already shown that this can be always achieved. The positivity of Eq. (15) is also satisfied if $\langle 0_N|\rho_{N-1}|0_N\rangle < \langle 1_N|\rho_{N-1}|1_N\rangle$ and

$$\sum_{i_{N+1}=0}^{1} |\langle 1_N|i_{N+1}\rangle|^2 |\langle i_{N+1}|\zeta\rangle|^2 > |\langle 1_N|\zeta\rangle|^2, \tag{18}$$

which has the same meaning as the (17) condition, starting from the $|1_{N+1}\rangle$ state instead of from $|0_{N+1}\rangle$.

Thus, we have shown that the probability of success can be increased by adding suitable observables to the process. Since each suitable θ_i observable depends on

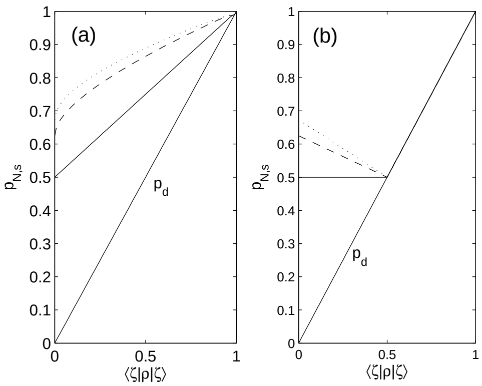

FIG. 3: Maximum value of $p_{N,s}$ probability as a function of the $\langle\zeta|\rho|\zeta\rangle$ initial probability for different N values: $N = 1$ (solid), $N = 2$ (dash), $N = 3$ (dot), for: (a) an initial pure state $\gamma = 1$ and (b) a mixed initial state $\gamma = 0$. The diagonal solid line corresponds to p_d.

the ρ initial state and the $|\zeta\rangle$ target, we can conclude that as N goes to infinity, the fidelity and the probability of finding $|\zeta\rangle$ will go to 1. This conclusion also can be obtained by studying the Hilbert-Schmidt distance [23] between ρ_N and $|\zeta\rangle\langle\zeta|$.

Figure 3 shows the results of a numerical simulation which finds the (13) probability for a given set of bases $\{|0_j\rangle, |1_j\rangle\}$ $(j = 1, 2, \ldots N)$ with N and ρ fixed. On it is plotted the maximum value of $p_{N,s}$ as a function of the $\langle\zeta|\rho|\zeta\rangle$ initial probability for different N values: $N = 1$ (solid), $N = 2$ (dash), $N = 3$ (dot), when (a) $\gamma = 1$ and (b) $\gamma = 0$. In Fig. 3.a, which corresponds an initial pure state, we can see that $p_{N,s}$ increases with respect to p_d (the diagonal) for all $\langle\zeta|\rho|\zeta\rangle$ initial probability. In Fig. 3.b, which corresponds to an initial mixed state (diagonal in the $\{|\zeta\rangle, |\zeta_\perp\rangle\}$) basis), we can see that $p_{N,s}$ increases with respect to p_d (the diagonal) only for $\langle\zeta|\rho|\zeta\rangle < 1/2$.

IV. CONCLUSIONS

In summary, we have studied the problem of driving a known initial quantum state onto a known pure state without using any unitary transformation. This task can be achieved by means of von Neumann measurement processes, introducing N observables which are consecutively measured in order to get the state closer to the target state. We proved that the probability of projecting onto the target can be increased by adding suitable observables to the process. Since each of these suitable observable depends on the ρ initial state and on the $|\zeta\rangle$ target, we conclude that as N increase the probability of finding $|\zeta\rangle$ goes to 1.

For a physical implementation of the above described process one could address the problem of keeping the initial flux of a beam composed of a collection of systems in the same state, each one exposed to a postselection-measurement procedure. For instance, let us consider a source of monochromatic and vertically linear polarized photons [24]. In order to obtain photons in a horizontally linear polarized state it is required to put a linear polarizer in their path. Implementing two linear polarizers in a suitable configuration, the outcome flux with horizontal linear polarization is decreased fifty per cent with respect to the incoming flux. By implementing more than two linear polarizers, as is suggested above, the output flux of the beam can be increased meaningfully and it can be approached to the initial flux, depending on the number of linear polarizers arranged suitably. Since in this scheme only one component of the each linear polarized flux contributes to the success probability, it will converge a little more slowly than our protocol, however it will also go to 1 as the number of linear polarizers arranged suitably increases, preserving approximately the initial flux. A nonlinear crystal can change the polarization of a photon while preserving the flux; however, it also preserves the initial mix degree. In our scheme, independently of the initial mix degree, the output is pure.

Further studies could be generalized considering a d-dimensional Hilbert space.

This work was supported by Milenio Grant ICM P02-49F, and FONDECYT Grants 1030671, 1040591, and 1040385.

[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, U.K., 2000); G. Alber et al. *Quantum Information* (Springer, Berlin, 2001).
[2] R. Landauer, Phys. Lett. A **217**, 188 (1996).
[3] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[4] M. Żukowsky, A. Zeilinger, M. A. Horne, and A. K. Ekert, Phys. Rev. Lett. **71**, 4287 (1993).
[5] W. K. Wootters and W. H. Zurek, Nature **299**, 802 (1982).
[6] L. M. Duan and G. C. Guo, Phys. Rev. Lett. **80**, 4999 (1998).
[7] A. K. Pati and S. L. Braunstein, Nature (London) **404**, 164 (2000).
[8] Z. Zhao, Y. A. Chen, A. N. Zhang, T. Yang, H. J. Briegel, and J. W. Pan, Nature (London) **430**, 54 (2004).
[9] J. W. Pan, M. Daniell, S. Gasparoni, G. Weihs, and A. Zeilinger, Phys. Rev. Lett. **86**, 4435 (2001).
[10] J. W. Pan, D. Bouwmeester, M. Daniell, H. Weinfurter, and A. Zeilinger, Nature (London) **403**, 515 (2000).

[11] D. Bouwmeester, J. W. Pan, M. Daniell, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. **82**, 1345 (1999).

[12] D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu, Phys. Rev. Lett. **80**, 1121 (1998).

[13] D. Bouwmeester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature (London) **390**, 575 (1997).

[14] A. Lamas-Linares, C. Simon, J. C. Howell, and D. Bouwmeester, Science **296**, 712 (2002).

[15] F. De Martini, V. Bužek, F. Sciarrino, and C. Sias, Nature (London) **419**, 815 (2002).

[16] M. Ziman, P. Štelmachovič, V. Bužek, M. Hillery, M. Scarani, and N. Gisin, Phys. Rev. A **65**, 042105 (2002).

[17] A. Barenco, C. H. Bennett, R. Cleve, D. P. DiVincenzo, N. Margolus, P. Shor, T. Sleator, J. A. Smolin, and H. Weinfurter, Phys. Rev. A **52**, 3457 (1995).

[18] S. G. Schirmer and J. V. Leahy, Phys. Rev. A **63**, 025403 (2001); F. Albertini and D. D. Alessandro, quant-ph/0106128.

[19] L. Roa, A. Delgado, M. L. Ladrón de Guevara, and A. B. Klimov, Phys. Rev. A **73**, 012322 (2006).

[20] Luis Roa and G. A. Olivares-Rentería, Phys. Rev. A. **73**, 062327 (2006).

[21] J. von Neumann, Ann. Math. **32**, 191 (1931); J. von Neumann, *Mathematische Grundlagen der Quantenmechanik* (Springer, Berlin, 1932).

[22] "To approach the state of the system to the $|\zeta\rangle$ target state"can be understood in the sense that the fidelity or probability to obtain the $|\zeta\rangle$ state goes closer to 1.

[23] J. Lee, M. S. Kim, and Č. Brukner, Phys. Rev. Lett. **91**, 087902 (2003).

[24] Asher Peres, *Quantum Theory: Concepts and Methods* (Kluwer Academic Publishers, 1998).